Longitudinal instabilities in the Accumulator for µ2e project

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List of Main Parameters

1. Project beam intensity:	$N = 3 \times 0.5\text{E}13 = 1.5\text{E}13$
2. Project beam current:	$J = 3 \times 0.5$ A = 1.5 A
3. Shunt impedance of cavities:	$R = 7 \times 50$ kOhm = 350 kOhm
4. Central orbit circumference:	$C = 474.1$ m
5. Central revolution frequency:	$f_0 = 0.629$ MHz
6. Central beam energy:	$E = 9.55 \times 0.93826 = 8.96$ GeV
7. Slippage factor:	$\eta = 0.01456$
8. Space between the batches:	$\Delta E = 12.5$ MeV
9. Corresponding frequency shift:	$\Delta f/f \approx 2 \times 10{-}5$
10. Rms energy spread of the butch:	$\Delta E = 1.5$ MeV

Longitudinal Instabilities

For the bunching scenario in the Accumulator for m2e experiment, the beam batch from the Booster (and then from Recycler) is injected onto the top (injection) orbit of the Accumulator Ring. Then, the first batch moves from the top orbit, in the nominal central orbit and after that on the bottom orbit. Consequently, the 2nd and third batches will correspondingly occupy the central and the top orbits.

For the written above design parameters, the beam loading voltage, induced in the RF cavity will be of 1.5A×350 kOhm=525 kV for the total beam current of all three batches. But even for each single batch the current of 0.5A, still induces a loading voltage of 175 kV, that is a very heavy beam loading factor.

Longitudinal instabilities due to beam loading we have predicted analytically. In our studies we have observed instabilities numerically, considering only a single beam batch, circulating in the Accumulator Ring in the injection (top) orbit and made analytical evaluations of the instability growth rates.

In our numerical model, the beam loading is calculated by formula:

$$V(t) = JR[C_4 \cos(2\pi f t) + S_4 \sin(2\pi f t)]$$

where C_4 and S_4 are normalized harmonics of the beam current including sum over all particles:

$$C_4 = \frac{2}{N}\sum_{n=1}^{N} \cos(2\pi f t_n), \quad S_4 = \frac{2}{N}\sum_{n=1}^{N} \sin(2\pi f t_n)\ ,$$

The results of simulations are presented in the pictures below (phase space: time/revolution time × 2π, energy deviation (GeV)).

Fig. 1 shows longitudinal beam dynamics for a uniformly-populated initial phase distribution. Figs. 2, 3 show longitudinal dynamics of the initially notched distributions, with 2% and 10% azimuthal density gaps correspondingly. All cases demonstrated instabilities development, with higher instability growth rates for notched beams.

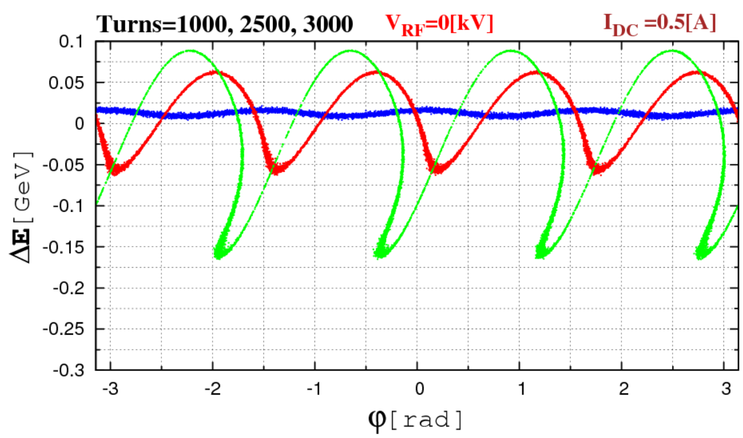

Fig. 1 One batch with full current J=0.5 A on the injection orbit after (1, 2.5, 3)×10^3 turns (blue, red, green) for an uniform initial distribution (zero notched).

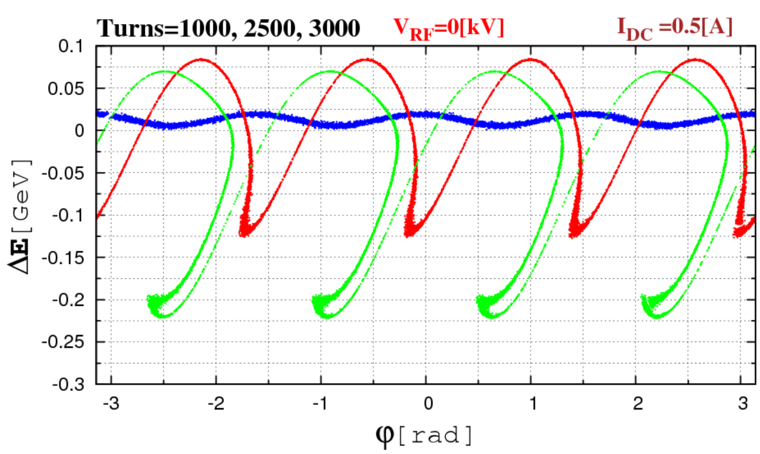

Fig. 2 One batch with full current J=0.5 A on the injection orbit after (1, 2.5, 3)×10^3 turns (blue, red, green) for a 2% notched initial distribution.

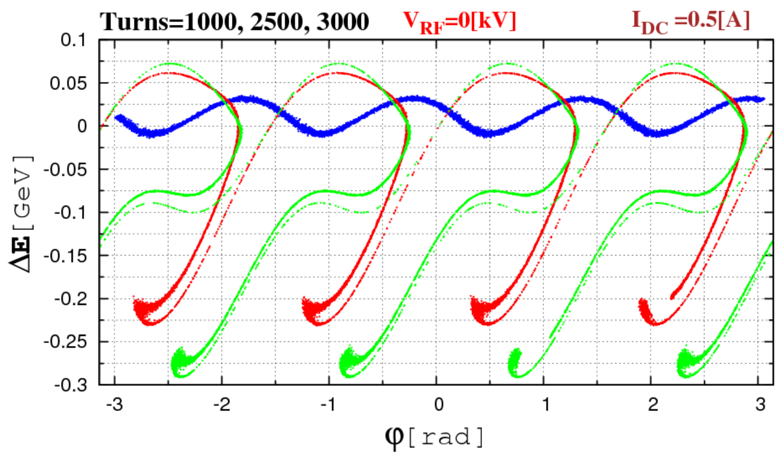

Fig. 3 One batch with full current J=0.5 A on the injection orbit after (1, 2.5, 3)×10^3 turns (blue, red, green) for a 10% notched initial distribution.

One can see an inappropriate energy spread of the batch already after 1-2 thousand turns due to instabilities.

So far we have been considering a single batch dynamics, placed on the injection orbit. On the central and the bottom orbits a similar instability growth occurs. In the figures below we always depict the batches on the different orbits, combined in one picture, although the simulation was performed for each batch separately.

If we decrease beam currents, the instabilities will still develop, slower though. Fig. 4 shows longitudinal dynamics after 10^4 and 1.5×10^4 turns for the beam current of 0.06A (12% from the nominal current of 0.5A).

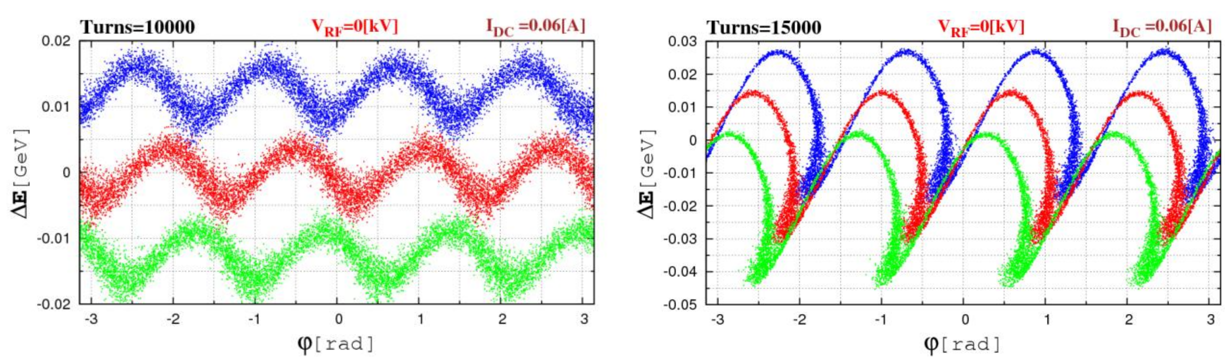

Fig. 4 Three batches on the injection, central and bottom orbits (blue, red and green) after 10^4 turns (left) and 1.5×10^4 turns (right) for the beam currents of 0.06A (12% from 0.5A).

For a lower current the instability growth rate is lower, as shown in Fig. 5, but still may cause problems after more turns.

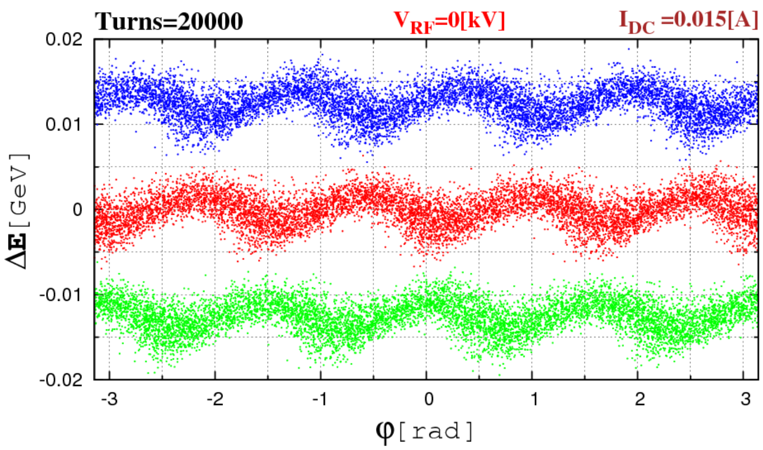

Fig. 5 Three batches on the injection, central and bottom orbits (blue, red and green) after 2×10^4 turns for the beam currents of 0.015A (3% from 0.5A).

For a 3% of the design current, as shown in Fig. 5, a beam batch becomes disturbed after 2×10^4 turns, but looks still acceptable. It is important therefore to evaluate the thresholds of the longitudinal instability.

Threshold of Instability

A coasting beam is unstable if its characteristic point is located inside the curve plotted in Fig. 6. The characteristic point is:

$$Y_k = \frac{2\pi i k \eta E}{e\beta^2 J_0 Z_k}\left(\frac{\sigma_E}{E}\right)^2 = \frac{0.93i}{JZ_4(\mathrm{kV})} \tag{1}$$

where k is harmonic number of the beam wave, Z_k is beam coupling impedance of this harmonic. Numerical expression is obtained at k = 4 with the Accumulator parameters taken from the list of parameters above.

Cavity Impedance

With the RCL model applied, the impedance of a cavity at the frequency f depends is:

$$Z(f) = \frac{R}{1+iQ(f_c/f - f/f_c)} \tag{2}$$

where f_c is the cavity eigenfrequency, Q is the quality factor.

Only the multiples of a revolution frequency which slightly differ for different particles, are essential. At $f \approx 4f_c$, the impedance is almost real value R = 350 kΩ. Theoretical instability threshold of the Accumulator is 3.5 mA at these conditions. The region $f \simeq f_c$

Instability Threshold Without Feedback

For the above-listed set of parameters, $f_c \approx f$, gives a dominating contribution at Q=125 ≫ 1, and for the centers of the upper, the middle, and the lower batches:

$$\frac{f}{f_c} = \frac{k}{4}\left(1 + \frac{\eta \Delta E}{E}\right) = \frac{k}{4}[1 + \{-2, 0, 2\} \times 10^{-5}]$$

The case of interest is k = 4 when $Z_4 = R$ = 350 kΩ almost in dependently on the particle revolution frequency (for comparison, $Z_5 = R/(1 - 56i) \approx -6i\ k\Omega$

Then we obtain:

$$Y_4 = \frac{2.66 \times 10^{-5} i}{J\ (\mathrm{A})}$$

A Gaussian beam is unstable with imaginary Y if |Y | < 0.76 that corresponds to J > 3.5 mA, or 0.7% of the project current of 0.5 A for a single batch. (A total current all three batches is 1.5 A).

Numerical simulations demonstrated that any batch with a lower, than 0.4% of the project intensity remains stable, regardless if the batch occupies the inner, the outer or a .

In Fig. 6 the longitudinal coordinates $(\Delta\varphi_k, \Delta E_k), k = 1, \dots, 25000$ of three batches with the currents of 0.4%, 0.7% and 1% from the project value, are shown after 10^5 turns. In each picture all 3 batches are pooled together just for illustration, whereas actually they were simulated separately.

The instability threshold for all three batches should slightly differ from that, calculated for each separate batch, due to a non-Gaussian distribution of the combined density.

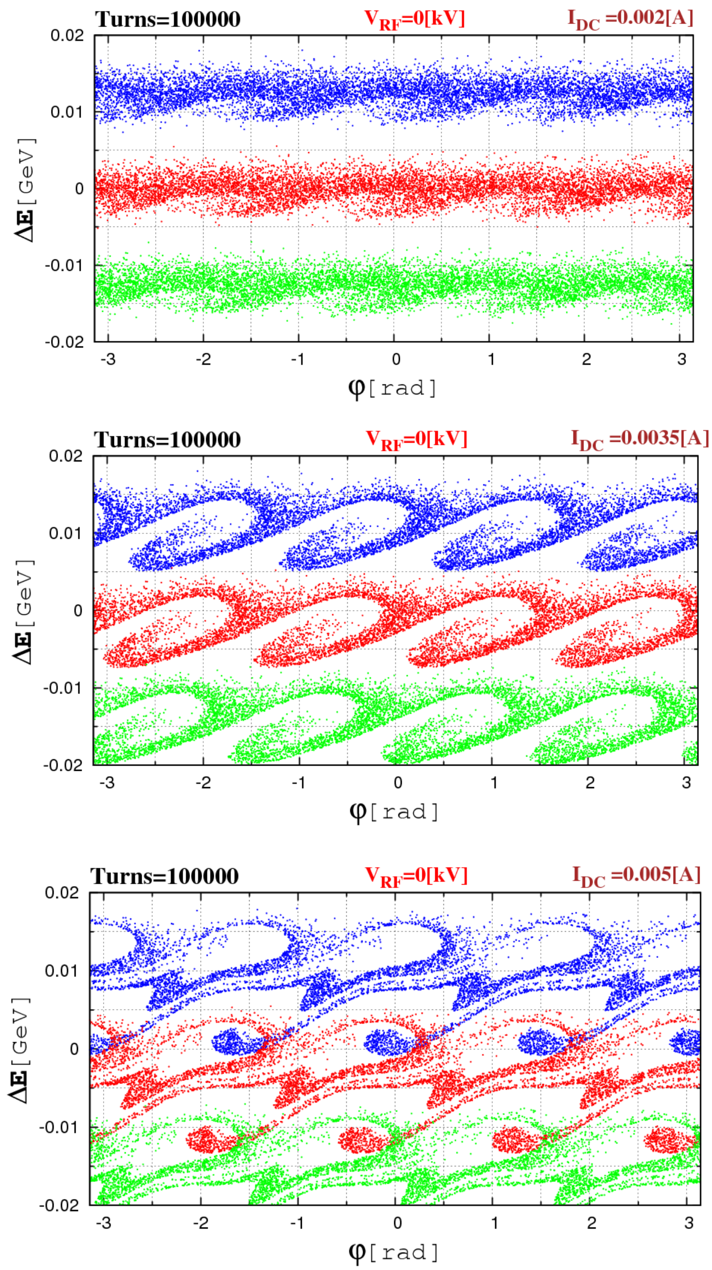

Fig. 7 Three batches on the injection, central and bottom orbits (blue, red and green) after 10^5 turns for the beam currents of 0.002A, 0.0035 and 0.005 A (0.4%, 0.7% and 1% from 0.5A).

The theoretically estimated stability threshold of 0.7% (3.5 mA) therefore, is rather close to the numerical results.

The conclusion is: to suppress instabilities and to accommodate a full beam current in the Accumulator Ring, we need to implement a feedback system.